Let's look at the concept of conditional probability in detail today. (As if the probability questions weren't tricky enough!) But since I like to discuss advanced concepts in this blog (in addition to alternative approaches and very important fundamentals), it would not be fair on my part to end the probability discussion without a quick review of conditional probability. Let me start by tossing a question at you.

Question 1: Alex tosses a coin four times. On two of the tosses (we don't know which two), he gets 'Heads'. What is the probability that he gets 'Tails' on other two tosses?

Solution: Wait a minute! Isn't it something like the Binomial Probability questions we saw last week? It is but notice that it is also a conditional probability question. You are given that on at least 2 tosses, he got 'Heads'. Under this condition, you want to find the probability that he got 2 tails i.e. he got 2 heads and 2 tails on his 4 tosses.

Conditional Probability is calculated as given below:

P(A given B) = P(A)/P(B)

Here, we are trying to find the probability that event A happens given that event B happens. To understand this formula, think of it this way:

Say there are a total of 100 cases and event B takes place in 10 cases. Also, event A takes place in 5 of the 10 cases in which event B takes place (A is a more restricted event under event B). Let's say we know that event B has taken place. This means that one of the 10 cases has occurred. The probability that A has taken place is 5/10 = 1/2 and not 5/100. I hope this makes sense to you. Let me take an example to make this clearer.

GMAT score can take one of 61 values (200/210/220 … 780/790/800). So there are a total of 61 cases. What is the probability that I will score above 700 on GMAT? (well, it should be 100% because otherwise I should not be writing blog posts on GMAT but let's assume that all the scores are equally likely)

There are 10 possible scores above 700 (710/720/730 … 800). Probability of a score above 700 = 10/61. That is our simple probability that we have been working on till date.

Now, consider this: You know that I scored above 600. How much exactly, you do not know! What will you say is the probability that I scored above 700? (again assuming that all the scores are equally likely)

I did score above 600. Now, what is the probability that I scored above 700? There are 20 possible scores above 600 (610/620/630 … 800). Any of them could have been my score. What is the probability that I actually scored above 700? It is 10/20. The event that I scored more than 700 is event A. It is more restrictive than event B i.e. the event that I scored more than 600. Given that event B took place i.e. I scored above 600, the probability that event A took place i.e. I scored above 700 is P(Score above 700)/P(Score above 600). This is conditional probability.

I hope you see the difference between probability and conditional probability.

Let's go back to the original question now.

We want to find this probability: P('2 Heads and 2 Tails' given 'At least 2 Heads') = P(2 Heads and 2 Tails)/P(At least 2 Heads)

We can easily find P(2 Heads and 2 Tails) and P(At least 2 Heads) since we are comfortable with the concepts of binomial probability! (right?)

P(2 Heads and 2 Tails) = (1/2)*(1/2)*(1/2)*(1/2) * 4!/(2!*2!) = 3/8

You multiply by 4!/(2!*2!) because out of the four tosses, any 2 could be heads and the other two would be tails. So you have to account for all arrangements: HHTT, HTHT, TTHH etc

P(Atleast 2 Heads) = P(2 Heads and 2 Tails) + P(3 Heads, 1 Tails) + P(4 Heads)

Let me remind you here that we can also find P(Atleast 2 Heads)  in the reverse way like this:

P(Atleast 2 Heads)  = 1 – [P(4 Tails) + P(3 Tails, 1 Heads)]

Let me show you the calculations involved in both the methods.

P(2 Heads and 2 Tails) = 3/8 (calculated above)

P(3 Heads, 1 Tails) = (1/2)*(1/2)*(1/2)*(1/2) * 4!/3! = 1/4

We multiply by 4!/3! to account for all arrangements e.g. HHHT, HHTH etc

P(4 Heads) = (1/2)*(1/2)*(1/2)*(1/2) = 1/16

P(Atleast 2 Heads)  = 3/8 + 1/4 + 1/16 = 11/16

OR

P(4 Tails) = (1/2)*(1/2)*(1/2)*(1/2) = 1/16

P(3 Tails, 1 Heads) = (1/2)*(1/2)*(1/2)*(1/2) * 4!/3! = 1/4

P(Atleast 2 Heads)  = 1 – (1/16 + 1/4) = 11/16

As expected, the value of P(Atleast 2 Heads)  is the same using either method.

P('2 Heads and 2 Tails' given 'At least 2 Heads') = P(2 Heads and 2 Tails)/P(At least 2 Heads) = (3/8)/(11/16) = 6/11

Notice here that you can ignore all the (1/2)s since in every case, you get (1/2)*(1/2)*(1/2)*(1/2) because Heads and Tails have equal probability. You can simply solve this question using this method:

No of arrangements with 2 Heads and 2 Tails = 4!/(2!*2!) = 6

No of arrangements with 3 Heads and 1 Tails = 4!/3! = 4

No of arrangements with 4 Heads = 4!/4! = 1

No of arrangements with at least 2 Heads = 6 + 4 + 1 = 11

P('2 Heads and 2 Tails' given 'At least 2 Heads') = 6/11

Out of the total number of arrangements of 'At least 2 Heads' (which is 11), only 6 are such that you get 2 Heads and 2 Tails.

Mind you, you cannot do that if the probabilities differ. Look at the question given below:

Question 2: Alex has five children. He has at least two girls (you do not know which two of his five children are girls). What is the probability that he has at least two boys too? (The probability of having a boy is 0.4 while the probability of having a girl is 0.6)

Think about what you are going to do here. We will look at the solution of this question next week.